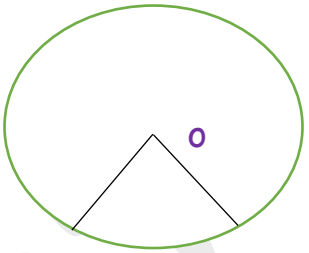
Exercise: 11.1

1. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Sol. Radius of circle = 6 cm



$\therefore$ Area of the sector = $\frac{\theta}{360^\circ}\pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 6 \times 6 \text{ cm}^2$

$= \frac{22}{7} \times 6 \text{ cm}^2$

$= \frac{132}{7} \text{ cm}^2$

Area of the sector making angle 60° is $\frac{132}{7}$ cm^2 Ans.

2. Find the area of a quadrant of a circle whose circumference is 22 cm.

Sol. Circumference of the circle = 22 cm

$2\pi r = 22 \text{ cm}$

$2 \times \frac{22}{7} \times r = 22$

$r = \frac{22 \times 7}{2 \times 22}$

$\therefore \quad r = \frac{7}{2} \text{ cm}$

$\therefore$ Area of the quadrant = $\frac{\theta}{360^\circ}\pi r^2$

$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{1}{4} \times \frac{11}{1} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{8} \text{ cm}^2$

$\therefore$ Area of the quadrant is $\frac{77}{8}$ cm^2 Ans.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Sol. Length of the minute hand of the clock is 14 cm

$\therefore$ Radius of the circle (r) = 14 cm

Angle rotated by minute hand in 1 hour = 360°

$\therefore$ Angle rotated by minute hand in 5 minutes = $\frac{360^\circ \times 5}{60^\circ}$

$= 30^\circ$

Area of the sector made by minute hand $= \frac{\theta}{360^\circ}\ \pi r^2$

$= \frac{30^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14\ \text{cm}^2$

$= \frac{1}{12} \times 22 \times 2 \times 14\ \text{cm}^2$

$= \frac{154}{3}\ \text{cm}^2$

Area swept by the minute hand in 5 minutes is $\frac{154}{3}\ \text{cm}^2$ Ans.

4. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment (ii) major sector. (Use $\pi = 3.14$)
Sol. Radius of the circle = 10 cm
Central angle = 90°
Area of minor segment = Area of the minor sector – area of ΔAOB

$= \frac{\theta}{360^\circ}\ \pi r^2 - \frac{1}{2}\ r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 - \frac{1}{2} \times 10 \times 10$

$= 78.5 - 50$

$= 28.5\ \text{cm}^2$

Angle in major sector = 360° - 90° = 270°
Area of the major sector $= \frac{\theta}{360^\circ}\ \pi r^2$

$= \frac{270^\circ}{360^\circ} \times 3.14 \times 10 \times 10$

$= 235.5\ \text{cm}^2$

$\therefore$ Area of minor segment is $28.5\ \text{cm}^2$

and area of the major segment is $235.5\ \text{cm}^2$ Ans.

5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
(i) the length of the arc
(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Sol. Radius of the circle = 21 cm

Central angle θ = 60°

(i) Length of the arc AB = $\frac{\theta}{360^\circ}\ 2\pi r = \frac{\theta}{180^\circ}\ \pi r$

$= \frac{60^\circ}{180^\circ} \times \frac{22}{7} \times 21$ cm

$= 22$ cm^2

The length of the arc is 22 cm.

(ii) Area of the sector $= \frac{\theta}{360^\circ}\ \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$ cm^2

$= 231$ cm^2

∴ Area of the sector formed by the arc is 231 cm^2

(iii) Area of segment = Area of the sector – area of equilateral ΔAOB

$= \frac{\theta}{360^\circ}\ \pi r^2 - \frac{\sqrt{3}}{4}\ r^2$

$= \left(\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21 - \frac{\sqrt{3}}{4} \times 21 \times 21\right)$ cm^2

$= \left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2

Area of the segment is $\left(231 - \frac{441\sqrt{3}}{4}\right)$ cm^2 Ans.

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and √3 = 1.73)

Sol. Radius of the circle = 15 cm

Central angle = 60°

Area of minor segment

= Area of the minor sector – area of equilateral ΔAOB

$= \frac{\theta}{360^\circ}\ \pi r^2 - \frac{\sqrt{3}}{4}\ r^2$

$= \frac{60°}{360°} \times 3.14 \times 15 \times 15 - \frac{1.73}{2} \times 15 \times 15$

$= 117.75 - 97.3125$

$= 20.4375 \text{ cm}^2$ .....(i)

Area of major segment = Area of circle – area of minor segment

$= \pi r^2 - \left(\frac{\theta}{360°} \pi r^2 - \frac{\sqrt{3}}{4} r^2\right)$

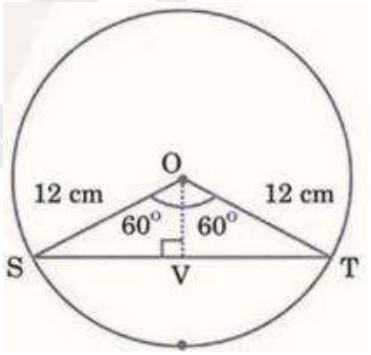
$= 3.14 \times 15 \times 15 - 20.4375 \text{ cm}^2$ from (1)

$= 706.5 - 20.4375 \text{ cm}^2$

$= 686.0625 \text{ cm}^2$

7. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Sol. The perpendicular OV drown form O to chord ST bisects ST.



$\therefore \text{SV} = \text{VT}$

In $\triangle$ OVS,

$\frac{OV}{OS} = \cos 60°$

$\Rightarrow \frac{OV}{12} = \frac{1}{2}$

$\Rightarrow OV = 6 \text{ cm}$

And $\frac{SV}{OS} = \sin 60°$

$\Rightarrow \frac{SV}{12} = \frac{\sqrt{3}}{2}$

$\Rightarrow OV = 6\sqrt{3} \text{ cm}$

$ST = 2 \times SV$

$= 2 \times 6\sqrt{3}$

$= 12\sqrt{3} \text{ cm}$

Area of $\triangle$ OST $= \frac{1}{2} \times ST \times OV$

$= \frac{1}{2} \times 12\sqrt{3} \times 6$

$= 36\sqrt{3}$

$= 36 \times 1.73$

$= 62.28 \text{ cm}^2$

Area of sector OSUT $= \frac{120^\circ}{360^\circ} \times \pi r^2$

$= \frac{1}{3} \times \pi(12)^2$

$= \frac{1}{3} \times 3.14 \times 12 \times 12$

$= 150.72 \text{ cm}^2$

Area of minor segment = Area of sector OSUT - Area of $\triangle$ OST

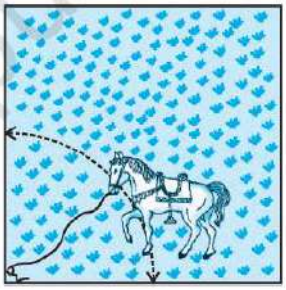
$= (150.72 - 62.28)\text{cm}^2$

$= 88.44 \text{ cm}^2$

8. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5m long rope (see Fig.). Find
(i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Sol. Side of square field = 15 m

Length of rope (radius of quadrant of circle) r = 5 m



(i) Area of quadrant = $\frac{\theta}{360^\circ} \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 5 \times 5 \text{ m}^2$

$= 19.625 \text{ m}^2$

∴ the area of that part of the field in which the horse can graze

$= 19.625 \text{ m}^2$

(ii) Area of circle if the length of rope is increased to 10 m

$= \frac{\theta}{360^\circ} \pi r^2$

$= \frac{90^\circ}{360^\circ} \times 3.14 \times 10 \times 10 \text{ m}^2$

$= 78.5 \text{ m}^2$

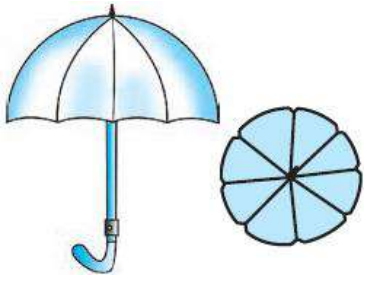
∴ Increase in grazing area = 78.5 m² - 19.625 m²

= 58.875 m² Ans.

9. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown in Fig. Find:
(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

Sol. Length of diameter of circle = 35 mm
$\therefore$ Radius of circle = $\frac{35}{2}$ mm
(i) Total length of silver wire required
= Circumference of the circle + Length of 5 diameters
= $2\pi r + (5\times35)$ mm
= $2 \times \frac{22}{7} \times \frac{35}{2} + 175$ mm
= 110 + 175 mm
= 285 mm
(ii) Number of sectors = 10
Area of each sector = $\frac{1}{10}$ area of circle
= $\frac{1}{10}\pi r^2$
= $\frac{1}{10} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2}$
= $\frac{385}{4}$ mm^2

10. An umbrella has 8 ribs which are equally spaced (see Fig.). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Sol. Number of ribs in umbrella = 8
Radius of umbrella while flat circle = 45 cm
Area between the two consecutive ribs of the umbrella
= $\frac{1}{8}$ area of circle
= $\frac{1}{8}\pi r^2$
= $\frac{1}{8} \times \frac{22}{7} \times 45 \times 45$
= $\frac{22275}{28}$ cm^2
$\therefore$ Area between the two consecutive ribs is $\frac{22275}{28}$ cm^2

11. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Sol. Radius of wiper (r) = 25 cm

Angle of the sector of circle made by wiper = 115°

Area of the sector made by each wiper

$= \frac{\theta}{360°} \pi r^2$

$= \frac{115°}{360°} \times \frac{22}{7} \times 25 \times 25 \text{ cm}^2$

$= \frac{158125}{252} \text{ cm}^2$

∴ the total area cleaned at each sweep of the blades

$= 2 \times \frac{158125}{252} \text{ cm}^2$

$= \frac{158125}{126} \text{ cm}^2$ Ans.

12. To warn ships for underwater rocks, a lighthouse spreads a red colored light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

Sol. Distance over which light spread i.e. radius, r = 16.5 km

$= \frac{33}{2}$ km

Central angle θ = 80°

Area of the sea over which the ships are warned = Area of the sector.

$= \frac{\theta}{360°} \pi r^2$

$= \frac{80°}{360°} \times 3.14 \times \frac{33}{2} \times \frac{33}{2} \text{ km}^2$

$= \frac{1709.73}{9} \text{ km}^2$

$= 189.97 \text{ km}^2$

the area of the sea over which the ships are warned is 189.97 km^2

13. A round table cover has six equal designs as shown in Fig. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2. (Use √3 = 1.7)

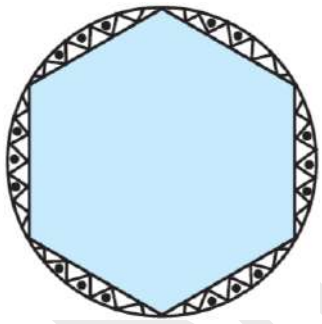
Sol. Number of equal designs in the table cover= 6

Radius of round table cover = 28 cm

Central angle = $\frac{360^\circ}{6}$ = 60°

Area of a design = Area of sector ACB - Area of equilateral ΔAOB

= $\frac{\theta}{360^\circ}\ \pi r^2 - \frac{\sqrt{3}}{4}\ r^2$



= $\frac{60^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28 - \frac{1.7}{4} \times 28 \times 28$

= 410.66 – 333.2 cm^2

= 77.46 cm^2

Area of 6 designs = 6 × 77.46 cm^2

= 464.76 cm^2

Cost of making design at ₹ 0.35 per cm^2 = ₹ 0.35 × 464.76 = ₹ 162.66

Total cost of making design is ₹ 162.66 Ans.

14. Tick the correct answer in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

(A) $\frac{p}{180}$ × 2πR (B) $\frac{p}{180}$ × π R^2 (C) $\frac{p}{720}$× 2π R^2 (D) $\frac{p}{360}$ × 2πR

Sol. Area of the sector made by each wiper = $\frac{\theta}{360^\circ}\ \pi r^2$

= $\frac{p}{360^\circ}\ \pi\ R^2$

= $\frac{p}{720^\circ}\ 2\ \pi\ R^2$

∴ Option (D) is correct.